

Hyperelastic behavior of two rubber materials under quasistatic and dynamic compressive loadings – testing, modeling and application^{*})

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Abstract: The mechanical properties of two rubber materials, RB-55 rubber and FM-32 foam rubber, were tested under quasistatic and dynamic compressive loadings with a universal testing machine and a non-metallic split Hopkinson pressure bar (SHPB), respectively. The results show that the hyperelasticity dominates the mechanical characteristics of the both materials. And the strain rate dependencies can be observed over the wide strain rate range from 10^{-2} s^{-1} to order 10^3 s^{-1} . But in the rather narrow bands of $10^{-2} - 10^0 \text{ s}^{-1}$ and $2 \cdot 10^3 - 6 \cdot 10^3 \text{ s}^{-1}$, the strain rate effects are not significant. In order to numerically simulate rod-explosive loading tests where the two rubber materials were used as a combined buffer, the strain rate-independent hyperelastic behaviors at the strain rate of order 10^3 s^{-1} were characterized by Ogden constitutive models, incompressible for RB-55 rubber and compressible for FM-32 foam rubber, respectively. The numerical prediction of the structural responses agrees very well with the experimental results. This means the testing and modeling are successful.

Keywords: rubber, hyperelastic behavior, compressive loading, constitutive model.

Właściwości hipersprężyste materiałów gumowych w warunkach kwazistatycznych i dynamicznych obciążeń ściskających – badania, modelowanie i zastosowanie

Streszczenie: Za pomocą uniwersalnej maszyny wytrzymałościowej oraz metodą pręta Hopkinsona (SHPB) zbadano właściwości mechaniczne gumy (RB-55) i gumy piankowej (FM-32) poddanych kwazistatycznym i dynamicznym obciążeniom ściskającym. Wyniki badań wskazały, że w materiałach tych dominują właściwości hipersprężyste. Krzywe zależności naprężenia od odkształcenia wyznaczono w szerokim zakresie szybkości odkształcania rzędu $10^{-2} - 10^3 \text{ s}^{-1}$. Stwierdzono, że szybkość odkształcania nie wywiera istotnego wpływu na krzywe naprężenie-odkształcenie tylko w dość wąskich zakresach wartości, tj. $10^{-2} - 0 \text{ s}^{-1}$ oraz $2 \cdot 10^3 - 6 \cdot 10^3 \text{ s}^{-1}$. W celu numerycznego symulowania testów obciążeniowych metodą wybuchających prętów (z ang. *rod-explosive loading tests*), gdzie oba materiały gumowe pełniły rolę buforów, zastosowano model konstytutywny Ogdena, przeznaczony do analizy materiałów hipersprężystych. Wyniki symulacji numerycznych wykazały dobrą zgodność z wynikami doświadczalnymi.

Słowa kluczowe: guma, właściwości hipersprężyste, obciążenie ściskające, model konstytutywny.

Rubber or rubber-like materials have been widely used to mitigate damage caused by shock loadings because of their low modulus, high damping capacity and large extensibility. In those applications, their mechanical properties are important for modeling and simulation, so

that better effects of isolation and/or energy absorption can be obtained.

In the above mentioned problems, the mechanical behavior refers to large strain, high strain rate and non-linear material response [1]. But up to now, most of the relevant investigations, such as the work by Dvorak *et al.* [2], Findik *et al.* [3], Ronan *et al.* [4] and Abu-Abdeen [5], have been limited in quasi static, creep, relaxation, small strain or linear problems. Recently, a number of experimental studies were published, referring to high strain rate compressive behavior of rubber or rubber-like materials [6–13]. Furthermore, there have been a number of corresponding constitutive models proposed for this kind of materials [14], including such models like: Blatz-Ko [15], Mooney-Rivlin [16, 17], Ogden [18, 19], *etc.*

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These provide a foundation for further researches and applications of dynamic constitutive relationship of rubber materials.

However, the mechanical properties of rubber materials are very sensitive to many factors, such as ingredients, operation parameters, environmental and loading conditions. Thus, there is neither a universal model nor a set of material parameters to describe them. The best countermeasure is to test the mechanical properties aiming at a specific rubber material under the conditions corresponding to the physical and mechanical environment, then select a right constitutive model and determine the material parameters.

In this paper two materials, RB-55 rubber and FM-32 foam rubber were investigated. First the quasistatic and dynamic compressive tests were performed. Then according to the test data, the mechanical behavior at the strain rate of order 10^3 s^{-1} was characterized by the Ogden constitutive models. Finally, an application example the two materials are used where as a combined buffer to shape the impulses generated by rod-explosive [20, 21] is given to illustrate the validity of the dynamic constitutive models.

EXPERIMENTAL PART

Materials

The two materials are common standard industrial rubber materials. The first one is RB-55 rubber supplied by Northwestern Rubber Company (China) characterized by the Shore hardness 55 and density 1.5415 g/cm^3 . It is a kind of white vacuum rubber made from natural rubber material with vulcanization technology. The RB-55 specimens are cylindrical with the size $\Phi 10 \times 3.54 \text{ mm}$, which were cut from a 3.54 mm thick plate. The second one is FM-32 foam rubber delivered by Chengdu Hongqi Rubber Company (China). It is a black open-cell sponge rubber material manufactured from natural rubber material with foaming technology. The foam rubber plates are also standard industrial products with varied mass densities. For the FM-32 material the Shore hardness was 32 and density 0.822 g/cm^3 . The FM-32 specimens are cylindrical with the size $\Phi 10 \times 3.5 \text{ mm}$. The photos of these two kinds of specimens are given in Fig. 1. Considering the fabrication error and material non-uniformities, the detailed sizes used in test data processing are according to final measurements.

Methods of testing

First, a series of quasistatic compressive tests were performed to obtain an elementary understanding of the mechanical properties. The specimens of the two rubber materials were tested under the compression strain rates 10^{-2} , 10^{-1} and 1 s^{-1} by a MTS RT/5 universal testing machine.

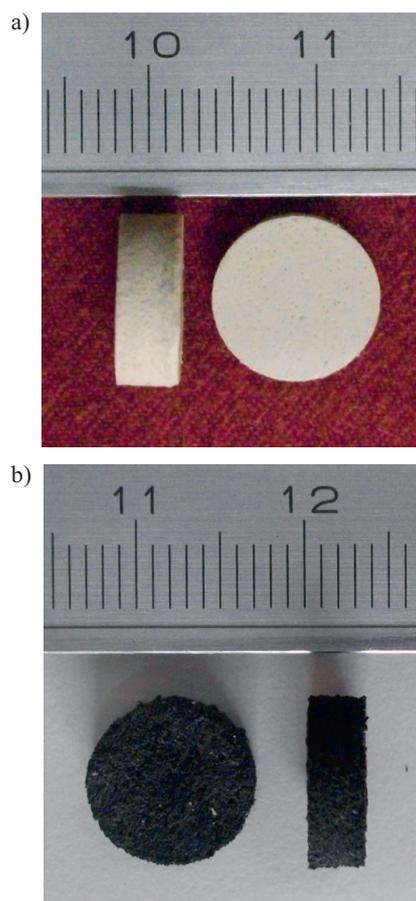


Fig. 1. Photographs of the specimens of the two materials: a) RB-55 rubber, b) FM-32 foam rubber

Then, a nonmetallic split Hopkinson pressure bar (SHPB) system was used to test the mechanical behavior under the strain rate between $2 \cdot 10^3 - 6 \cdot 10^3 \text{ s}^{-1}$.

The two rubber materials are both typical soft materials. General SHPB systems (metal bars based) can not be used to test their mechanical properties because the reflected strain waves are too weak to detect [6–13]. An effort utilizing high-sensitive semiconductor strain gauges was even made but it unfortunately failed. At last, a PMMA SHPB apparatus was selected after a series of trials and comparisons. Therein, the input/output bar and the impact bar (projectile) are all made of PMMA, and have the diameter of 12.5 mm. Their elastic modulus is 3 GPa, and the mass density is 1180 kg/m^3 . The length of the input/output bar is 1000 mm, and the impact bar is 30 mm long. Pulse shapers were used in the tests to adjust the input strain waveforms. The two interfaces at the ends of the specimens were enough lubricated before testing.

RESULTS AND DISCUSSION

Quasistatic tests

The tested engineering stress-strain curves are given in Fig. 2. From Fig. 2a, the obvious hyperelasticity can be

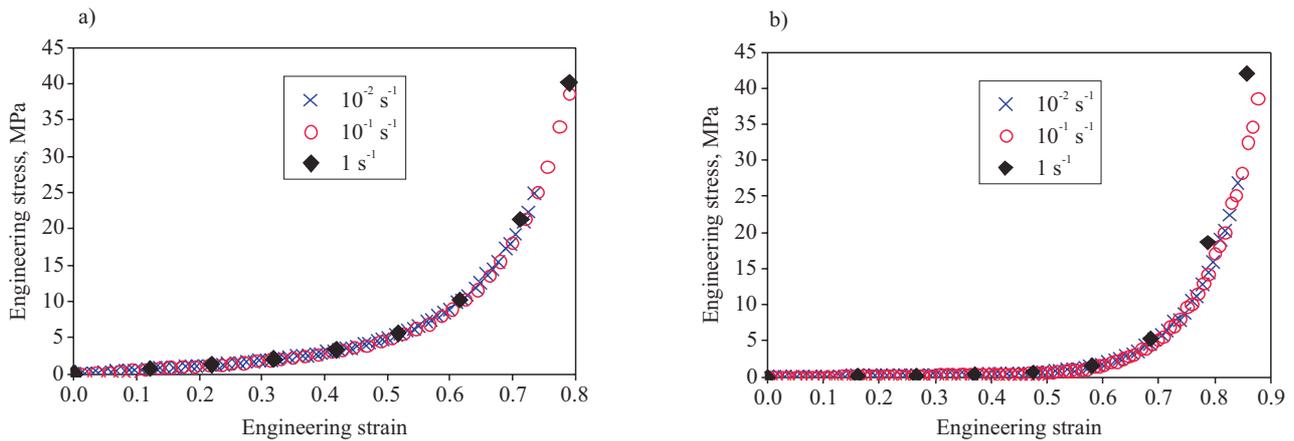


Fig. 2. Engineering stress-strain curves under quasistatic compressive loadings for: a) RB-55 rubber, b) FM-32 foam rubber

seen in the engineering stress-strain curves of RB-55 rubber. But under the different strain rates, the three curves nearly superposed together. So it can be drawn that the strain rate effect of the constitutive relationships of RB-55 rubber is not significant under quasistatic compressive loadings at the strain rate between 10^{-2} and 1 s^{-1} .

Similarly, for FM-32 foam rubber, the obvious hyperelasticity can be also found, as it was shown in Fig. 2b. But different from RB-55 rubber, this material is relatively soft in low strain range from 0 to 0.5, when the foam is not compacted into the solid state. Then, the curve slopes become higher and higher with the increase of the strain after it exceeds 0.5. This stage reflects the stress-strain relationship when the material is compacted into the solid state. As for the strain rate effect, the stress-strain curves under the strain rates 10^{-2} and 10^{-1} s^{-1} are almost consistent, but that below 1 s^{-1} is a little different from the other two. This phenomenon indicates that, under quasistatic compressive loadings (strain rate from 10^{-2} to 1 s^{-1}), the mechanical properties of FM-32 foam rubber are slightly strain rate-dependent. But for engineering problems, the slight strain rate dependence can be neglected.

To summarize, the quasistatic compressive tests show that the quasistatic mechanical properties distinctly indi-

cate hyperelasticity of both rubber materials. These properties are almost strain rate-independent.

Dynamic tests

In Fig. 3 the SHPB test results of the two materials under various strain rates are shown. As mentioned above, the specimen diameter is 10 mm and the input/output bar diameter is 12.5 mm. Thus, in the process of specimen compression, the radial expansion will let its diameter exceed the diameter of the input and output bars, when the test results become invalid. Therefore, Fig. 3 only gives the results with the limited strain range from 0 to 0.4. From Fig. 3a, we can find that the engineering stress-strain curves of RB-55 rubber have a slight rising trend, and show approximately linear relationship. But considering the rather large strain, the rising trend of the true stress-strain curves will be more significant. So the results shown in Fig 3a also show the hyperelastic behavior of RB-55 rubber. At the same time, the test results of FM-32 foam rubber given in Fig. 3b show a relatively strong hyperelastic property. Similarly to results presented in Fig. 2, in the two groups of results shown in Fig. 3, the curves of different strain rates are slightly different,

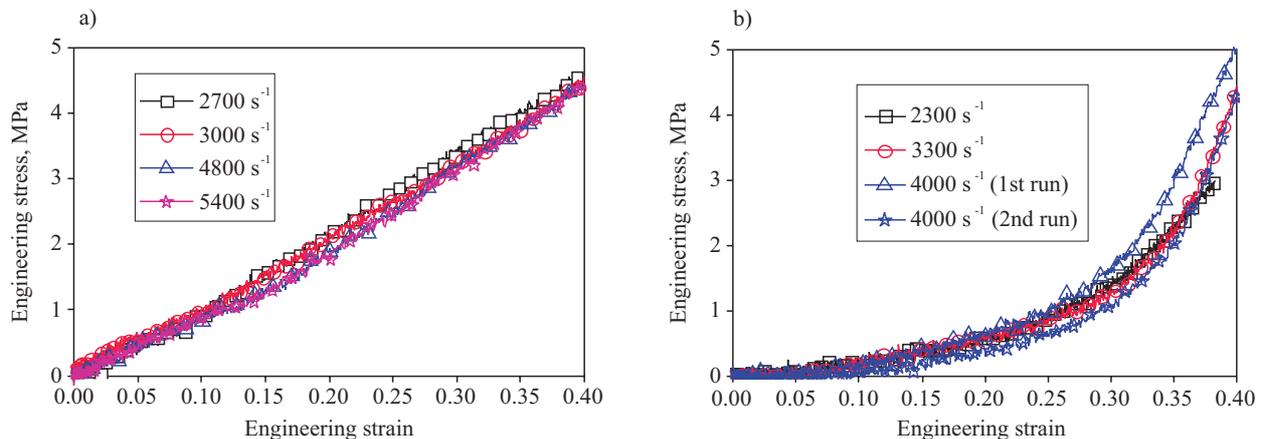


Fig. 3. Engineering stress-strain curves under dynamic compressive loadings for: a) RB-55 rubber, b) FM-32 foam rubber

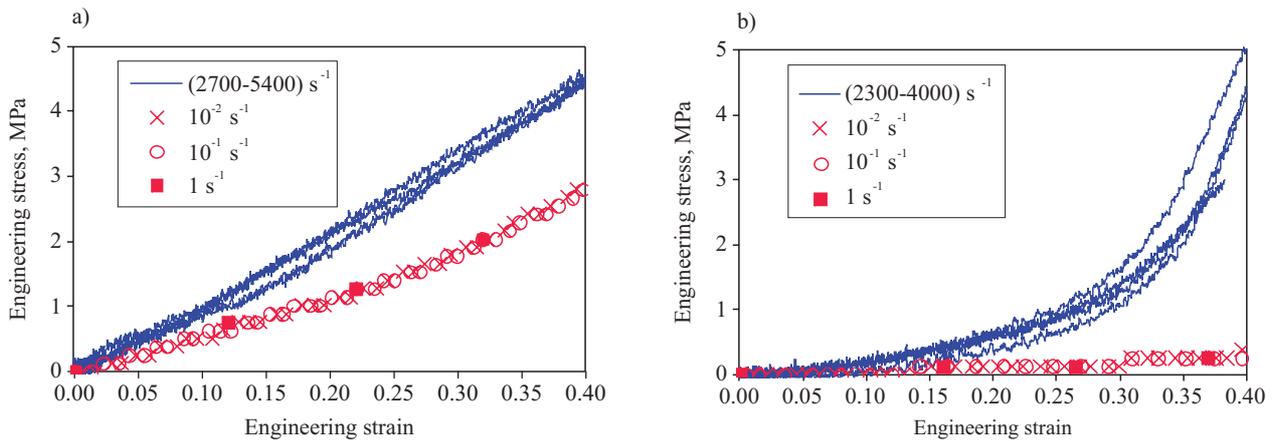


Fig. 4. Comparison between the quasistatic and dynamic mechanical properties of: a) RB-55 rubber, b) FM-32 foam rubber

but the strain rate effects have not a clear relationship. According to our analysis, the test error and dispersion are caused by various uncertainties, such as size error of the specimens, unflatness of the specimen ends and non-uniformities of the materials. These uncertainties just cover the essentially inconspicuous strain rate effects.

The quasistatic and dynamic results are partly compared in Fig. 4. It shows that the results under the strain rates less than 1 s^{-1} and of order 10^3 s^{-1} are clearly different, for the both materials. This indicates that the strain rate effects are significant when the loading rate widely ranges from 10^{-2} to order 10^3 s^{-1} . The tangent moduli at the engineering strain of 0.3 increase by approximately 50 and 1000 % for RB-55 rubber and FM-32 foam rubber, respectively. But as mentioned above, the strain rate effects are not significant over the strain rate from 10^{-2} to 1 s^{-1} and at rate of order 10^3 s^{-1} , respectively.

These trends agree with the previous experimental studies. By compressive tests of Adiprene L-100 rubber, Gray *et al.* [22] found that there was little change in constitutive behavior for the strain rate from 10^{-3} to 10^{-1} s^{-1} but the Young's modulus increased by 800 % with the strain rate increase from 10^{-3} to $3 \cdot 10^3 \text{ s}^{-1}$. Bergström and Boyce [23] tested the compressive properties of nitrile and chloroprene rubber and found that the compressive tangent modulus at a true strain of 0.45 increased by approximately 10 % with the strain rate increase from $2 \cdot 10^{-4}$ to 10^{-1} s^{-1} . And Lee *et al.* [24] found, for three unspecified rubber materials, the compressive tangent modulus increased by 400 % with the strain rate increase from 1 to $5 \cdot 10^3 \text{ s}^{-1}$. The above test data, both ours and the previous, show that the rate effect is not significant at low strain rates (generally below 1 s^{-1}), but it becomes very strong when the strain rate ranges from low level to high level (of order 10^3 s^{-1}), at least for some rubbers. Our test results also show that, the rate dependence is very slight at the strain rate from 10^{-2} to 1 s^{-1} or at the rate of order 10^3 s^{-1} .

Since the two materials can be regarded as strain rate-independent for dynamic problems with the strain rate of order 10^3 s^{-1} , the corresponding dynamic test results should be used to build or select appropriate corres-

ponding dynamic constitutive models and to determine the parameters to numerically simulate the impulsive loading tests where the strain rate is almost of order 10^3 s^{-1} .

Constitutive characterization

According to the above test results, the mechanical properties of the two materials confirm their hyperelasticity and are strain rate-independent under the strain rate of order 10^3 s^{-1} . Thus, strain rate-independent hyperelastic constitutive models can be used to characterize them. Presently, there are several hyperelastic constitutive models for rubber or rubber-like materials [14], such as Blatz-Ko model [15], Mooney-Rivlin model [16, 17] and Ogden model [18, 19]. Therein, the Blatz-Ko model is the simplest one, which is described by only one parameter, *i.e.* shear modulus [14], so its simulation precision is low. The Mooney-Rivlin model is the most widely used one, but it is not appropriate for strong strain-hardening characteristics. The Ogden model can simulate hyperelastic behavior in a relatively wide strain range and also can describe strong strain-hardening behavior [25–27]. The dynamic problem considered in this paper refers to both large strains induced by local explosion loadings and strong strain-hardening characteristics of the two rubbers. Consequently, the Ogden model is more appropriate.

In the Ogden model, the strain energy (W) is expressed as a function of the principal stretches [28]. For incompressible materials, the strain energy function is given by equation:

$$W = \sum_{i=1}^3 \sum_{j=1}^n \frac{\mu_j}{\alpha_j} (\lambda_i^{\alpha_j} - 1) = \sum_{j=1}^n \frac{\mu_j}{\alpha_j} (\lambda_1^{\alpha_j} + \lambda_2^{\alpha_j} + \lambda_3^{\alpha_j} - 3) \quad (1)$$

where: μ_j , α_j – the material parameters, n – the number of terms, λ_i ($i = 1, 2, 3$) – the principal stretches.

The following relationship is assumed:

$$\lambda_1 \lambda_2 \lambda_3 = 1 \quad (2)$$

For compressible materials, W function is expressed as:

$$W = \sum_{i=1}^3 \sum_{j=1}^n \frac{\mu_j}{\alpha_j} (\lambda_i^{\alpha_j} - 1) + K(J - 1 - \ln J) \quad (3)$$

where: λ_i ($i = 1, 2, 3$) — the principal stretches where the volumetric effects have been eliminated, K — the bulk modulus, J — the relative volume.

The parameters of eq. (3) should satisfy the following additional relationships:

$$J = \lambda_1 \lambda_2 \lambda_3 \quad (4)$$

$$\lambda_i^* = J^{-\frac{1}{3}} \lambda_i, i = 1, 2, 3 \quad (5)$$

$$\lambda_1^* \lambda_2^* \lambda_3^* = 1 \quad (6)$$

If the expression for W function is obtained, the three principal stresses can be formulated by the equations:

$$\sigma_1 = \frac{1}{\lambda_2 \lambda_3} \frac{\partial W}{\partial \lambda_1} \quad (7)$$

$$\sigma_2 = \frac{1}{\lambda_1 \lambda_3} \frac{\partial W}{\partial \lambda_2} \quad (8)$$

$$\sigma_3 = \frac{1}{\lambda_1 \lambda_2} \frac{\partial W}{\partial \lambda_3} \quad (9)$$

Thus, the material parameters can be determined according to the uniaxial compression test results by nonlinear curve fitting.

Table 1. Constitutive parameters of the two materials for the Ogden models ($n = 3$), where μ_j ($j = 1, 2, 3$) and α_j ($j = 1, 2, 3$) are material parameters varied for different materials

RB-55 rubber		FM-32 foam rubber	
parameter	value	parameter	value
α_1	-3.2000	α_1	-2.9600
α_2	0.7839	α_2	-1.7120
α_3	7.0960	α_3	4.9600
μ_1	-0.8856 MPa	μ_1	1.0360 MPa
μ_2	-3.7120 MPa	μ_2	0.4369 MPa
μ_3	0.0608 MPa	μ_3	2.2440 MPa

According to the above dynamic compression test results, the constitutive parameters for the Ogden models

($n = 3$) of the two materials were determined by nonlinear least square method, as given in Table 1. The engineering stress-strain curves predicted by the Ogden models are compared to the corresponding test results, as shown in Fig. 5.

Application

The constitutive models and parameters obtained in the above section have been successfully used in an application of testing dynamic structural responses [29, 30]. When a pulsed intense X-ray radiates on a structure, the instantaneous energy deposition will lead to a so-called blow-off impulse loaded on the structure surface. Then structural responses will be caused and may result in damage or failure of the structure. Several researchers experimentally simulate this kind of mechanical effects using the rod-explosive technique [20, 21].

The experimental configuration of the rod-explosive loading test is shown schematically in Fig. 6 [21]. The test item is a cylindrical steel shell with the diameter of 265 mm, length of 380 mm and thickness of 4 mm. Nineteen pentaerythritol tetranitrate (PETN) explosive rods (3 mm wide and 0.47 mm thick) were used to generate discrete impulsive loads to simulate the impulse induced by X-ray, which is approximately continuously distributed as a cosine function [21, 31]. A layer of RB-55 rubber (2.82 mm thick) and a layer of FM-32 foam rubber (8.14 mm thick) are combined as a buffer to shape the rod-explosive loads to be closer to a cosine distribution. For finite element simulation, the cylindrical shell, the FM-32 foam rubber layer and the RB-55 rubber layer were modeled firstly. Then the nineteen PETN explosive rods were modeled on the RB-55 rubber layer according to the practical distribution. An air layer 15.67 mm thick was also modeled to envelope the surfaces of the RB-55 rubber layer and the explosive rods to simulate the explosion fluid field. A numerical simulation [29, 30] was performed by the explicit finite software LS-DYNA to simulate the test where the Ogden constitutive models and corresponding parameters obtained in this paper were used.

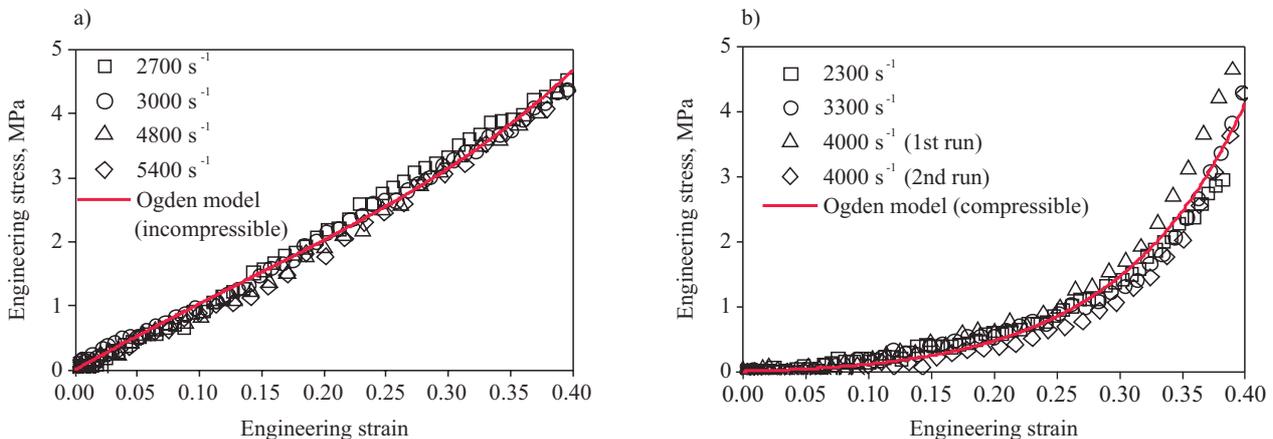


Fig. 5. Predicted engineering stress-strain curves compared to the dynamic test results for: a) RB-55 rubber, b) FM-32 foam rubber

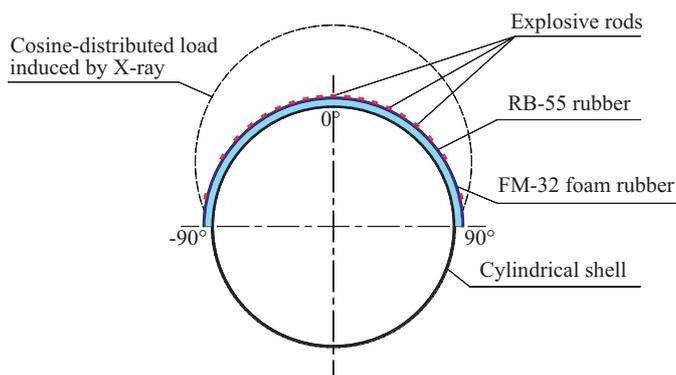


Fig. 6. Experimental configuration of the rod-explosive loading test (according to [21])

In the finite element simulation, the explosive rods exploded and generated a transient load. Then the loads were

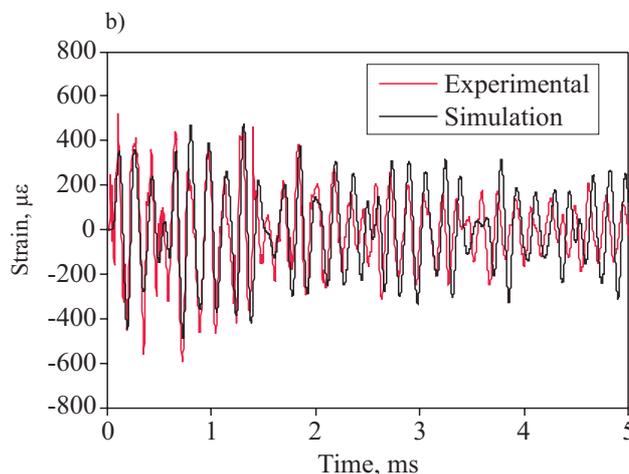
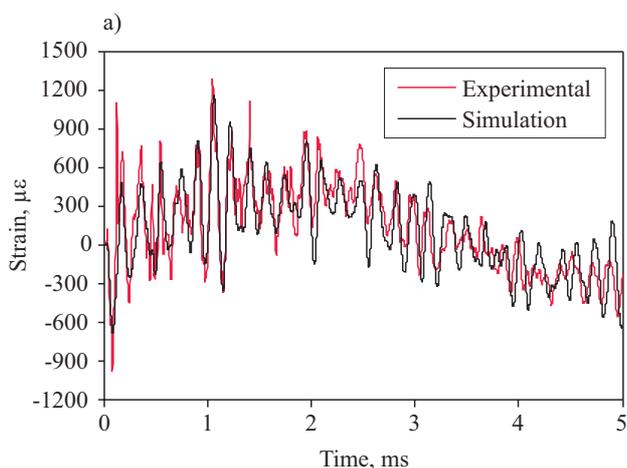


Fig. 7. Numerical and experimental strain histories on the inner surface at the angle of 0° (according to [30]): a) circumferential strain, b) axial strain

transmitted to the cylindrical shell and caused structural responses. The comparison between the numerical and experimental strain histories (both axial and circumferential) on the inner surface at 0° gives Fig. 7 [30].

This figure shows that the numerical results agree with the experimental in characteristics of both the amplitudes and frequencies, if the testing error is neglected. It is necessary to be pointed out that this kind of transient numerical prediction is very difficult and the above comparison is acceptable. This indicates that the testing and modeling of the two rubber materials are successful and acceptable for engineering applications.

CONCLUSIONS

— The hyperelasticity dominates the quasistatic (10^{-2} – 10^0 s^{-1}) and dynamic (of order 10^3 s^{-1}) mechanical properties of the two rubber materials (RB-55 and FM-32).

— The strain rate hardening is significant when the loading rate widely ranges from 10^{-2} to order 10^3 s^{-1} . But it is very slight in quasistatic (10^{-2} – 10^0 s^{-1}) and dynamic (of order 10^3 s^{-1}) compression tests, respectively.

— The strain rate-independent Ogden hyperelastic models (incompressible for RB-55 rubber and compressible for FM-32 foam rubber) were used to characterize the two materials and the parameters were determined according to the dynamic test results for numerical simulation of rod-explosive loading tests with the strain rate of order 10^3 s^{-1} in the two rubber materials.

— The numerical prediction of the structural responses induced by rod-explosive impulses and shaped by the two rubber materials agree very well with the experimental results. This indicates that the testing and modeling of the two rubber materials are successful and acceptable for engineering applications.

— The testing-modeling procedure and the results presented in this paper are essential for further researches and engineering applications.

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REFERENCES

- [1] Hoo Fatt M.S.H., Ouyang X.: *International Journal of Solids and Structures* **2007**, *44*, 6491. <http://dx.doi.org/10.1016/j.j-solstr.2007.02.038>
- [2] Dvorak E., Kelly M., McEntyre S., Goswami T.: *Materials & Design* **2003**, *24*, 397. [http://dx.doi.org/10.1016/S0261-3069\(03\)00053-0](http://dx.doi.org/10.1016/S0261-3069(03)00053-0)
- [3] Findik F., Yilmaz R., Köksal T.: *Materials & Design* **2004**, *25*, 269. <http://dx.doi.org/10.1016/j.matdes.2003.11.003>

- [4] Ronan S., Alshuth T., Jerrams S., Murphy N.: *Materials & Design* **2007**, 28, 1513. <http://dx.doi.org/10.1016/j.matdes.2006.03.009>
- [5] Abu-Abdeen M.: *Materials & Design* **2010**, 31, 2078. <http://dx.doi.org/10.1016/j.matdes.2009.10.006>
- [6] Rao S., Shim V.P.W., Quah S.E.: *Journal of Applied Polymer Science* **1997**, 66, 619. [http://dx.doi.org/10.1002/\(SICI\)1097-4628\(19971024\)66:4%3C619::AID-APP2%3E3.0.CO;2-V](http://dx.doi.org/10.1002/(SICI)1097-4628(19971024)66:4%3C619::AID-APP2%3E3.0.CO;2-V)
- [7] Yang L.M., Shim V.P.W., Lim C.T.: *International Journal of Impact Engineering* **2000**, 24, 545. [http://dx.doi.org/10.1016/S0734-743X\(99\)00044-5](http://dx.doi.org/10.1016/S0734-743X(99)00044-5)
- [8] Chen W., Lu F., Frew D.J., Forrestal M.J.: *Journal of Applied Mechanics* **2002**, 69, 214. <http://dx.doi.org/10.1115/1.1464871>
- [9] Song B., Chen W.: *Journal of Engineering Materials and Technology* **2003**, 125, 294. <http://dx.doi.org/10.1115/1.1584492>
- [10] Song B., Chen W.: *Journal of Engineering Materials and Technology* **2004**, 126, 213. <http://dx.doi.org/10.1115/1.1651097>
- [11] Song B., Chen W.: *Experimental Mechanics* **2004**, 44 (3), 300. <http://dx.doi.org/10.1007/BF02427897>
- [12] Shim V.P.W., Yang L.M., Lim C.T., Law P.M.: *Journal of Applied Polymer Science* **2004**, 92, 523. <http://dx.doi.org/10.1002/app.20029>
- [13] Quintavalla S.J., Johnson S.H.: *Rubber Chemistry and Technology* **2005**, 77 (5), 972. <http://dx.doi.org/10.5254/1.3547863>
- [14] Ali A., Hosseini M., Sahari B.B.: *American Journal of Engineering and Applied Sciences* **2010**, 3 (1), 232. <http://dx.doi.org/10.3844/ajeassp.2010.232.239>
- [15] Blatz P.J., Ko W.L.: *Transactions of The Society of Rheology* **1962**, 6, 223. <http://dx.doi.org/10.1122/1.548937>
- [16] Mooney M.A.: *Journal of Applied Physics* **1940**, 6, 582. <http://dx.doi.org/10.1063/1.1712836>
- [17] Holzapfel G.A.: “Nonlinear Solid Mechanics, A Continuum Approach for Engineering”, Wiley, New York 2001.
- [18] Ogden R.W.: *Proceedings of the Royal Society A* **1972**, A326, 565. <http://dx.doi.org/10.1098/rspa.1972.0026>
- [19] Ogden R.W.: “Non-linear Elastic Deformations”, Ellis Horwood Limited, Chichester 1984.
- [20] Rivera W.G., Benham R.A., Duggins B.D., Simmermacher T.W.: “Explosive Technique for Impulse Loading of Space Structures”, Sandia Technical Report, SAND-99-3175C, 1999.
- [21] Mao Y.J., Deng H.J., Li Y.L. *et al.*: *Applied Mechanics and Materials* **2010**, 29–32, 72. <http://dx.doi.org/10.4028/www.scientific.net/AMM.29-32.72>
- [22] Gray III G.T., Blumenthal W.R., Trujillo C.P., Carpenter R.W.: *Journal de Physique* **1997**, 7 (C3), 523. <http://dx.doi.org/10.1051/jp4:1997390>
- [23] Bergström J.S., Boyce M.C.: *Journal of the Mechanics and Physics of Solids* **1998**, 46 (5), 931. [http://dx.doi.org/10.1016/S0022-5096\(97\)00075-6](http://dx.doi.org/10.1016/S0022-5096(97)00075-6)
- [24] Lee O.S., Kim M.S., Kim K.J. *et al.*: *International Journal of Modern Physics B* **2003**, 17 (8–9), 1415. <http://dx.doi.org/10.1142/S0217979203019083>
- [25] Shergold O.A., Fleck N.A., Radford D.: *International Journal of Impact Engineering* **2006**, 32, 1384. <http://dx.doi.org/10.1016/j.ijimpeng.2004.11.010>
- [26] Miller K., Chinzei K.: *Journal of Biomechanics* **2002**, 35, 483. [http://dx.doi.org/10.1016/S0021-9290\(01\)00234-2](http://dx.doi.org/10.1016/S0021-9290(01)00234-2)
- [27] Jerrams S.J., Kaya M., Soon K.F.: *Materials & Design* **1998**, 19, 157. [http://dx.doi.org/10.1016/S0261-3069\(98\)00021-1](http://dx.doi.org/10.1016/S0261-3069(98)00021-1)
- [28] Hallquist J.O.: “LS-DYNA Theoretical Manual”, Livermore Software Technology Corporation, California 1988.
- [29] Mao Y.J., Li Y.L., Chen Y. *et al.*: *Chinese Journal of High Pressure Physics* **2012**, 26 (2), 155 (in Chinese). <http://dx.doi.org/10.11858/gywlxb.2012.02.006>
- [30] Mao Y.J., Li Y.L., Chen Y. *et al.*: *Chinese Journal of High Pressure Physics* **2013**, 27 (1), 76 (in Chinese). <http://dx.doi.org/10.11858/gywlxb.2013.01.011>
- [31] Mao Y.J., Li Y.L., Huang H.J.: *Advanced Materials Research* **2010**, 108–111, 1039. <http://dx.doi.org/10.4028/www.scientific.net/AMR.108-111.1039>

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